Quantum mechanics: phenomena and theory

Christian Wüthrich

http://www.wuthrich.net/

Introduction to Philosophy of Physics

The puzzle of quantum physics

John S Bell (2004, 173)

> *I think that conventional formulations of quantum theory, and of quantum field theory in particular, are unprofessionally vague and ambiguous. Professional theoretical physicists ought to be able to do better.*

John Stuart Bell ([2]2004). *Speakable and Unspeakable in Quantum Mechanics*. Cambridge: University of Cambridge Press.

Experiment 1: The cathode ray tube

The grainy nature of a fading cathode ray: particles?

Tim Maudlin (2019). *Philosophy of Physics: Quantum Theory*. Princeton University Press.

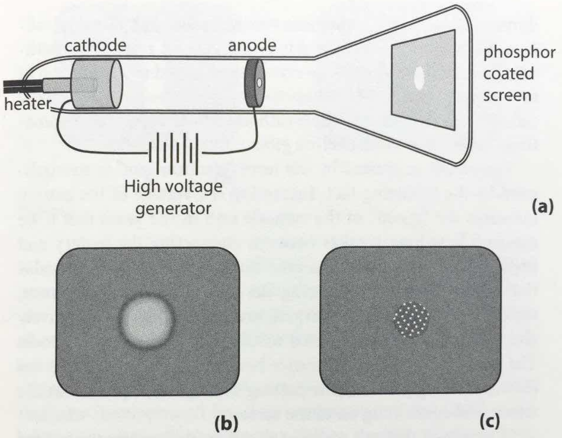


Figure 1

Experiment 2: The single slit

Diffraction at a narrow slit: waves?

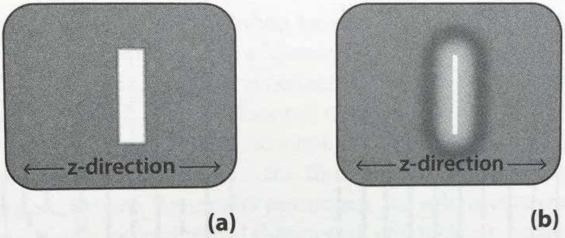


Figure 2

Experiment 2: The single slit

Reducing diffraction through shorter wavelength: wave-particle duality

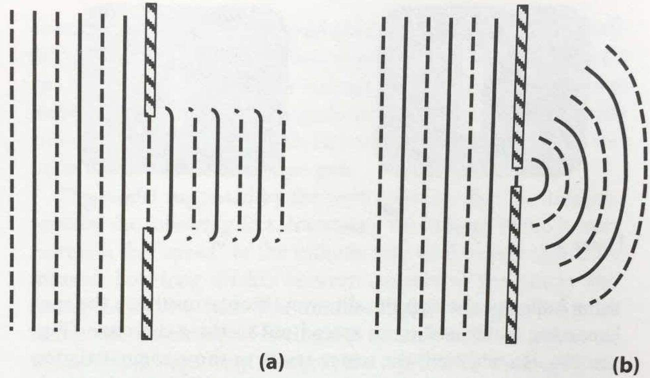


Figure 3

(solid lines: crests; dashed lines: troughs)

Experiment 3: The double slit

The quantum signature: interference and superposition

Richard Feynman *et al.* (1975, §37-1)

> *We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. We cannot explain the mystery in the sense of 'explaining' how it works. We will tell you how it works. In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics.*

Richard Feynman, Robert Leighton, and Matthew Sands (1975). *The Feynman Lectures of Physics*. Reading, MA: Addison-Wesley.

Experiment 3: The double slit

The quantum signature: crests and troughs

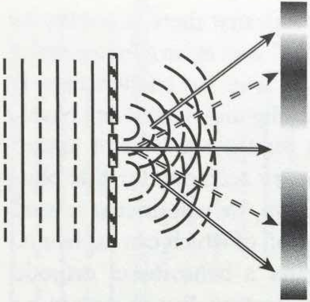


Figure 4

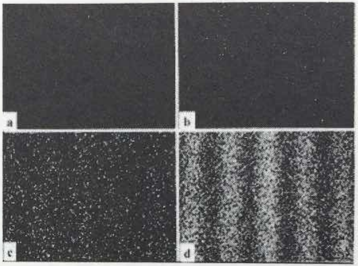


Figure 5. Credit: Reprinted courtesy of the Central Research Laboratory, Hitachi, Ltd., Japan.

Experiment 3: The double slit

How does an individual electron know to form an interference pattern?

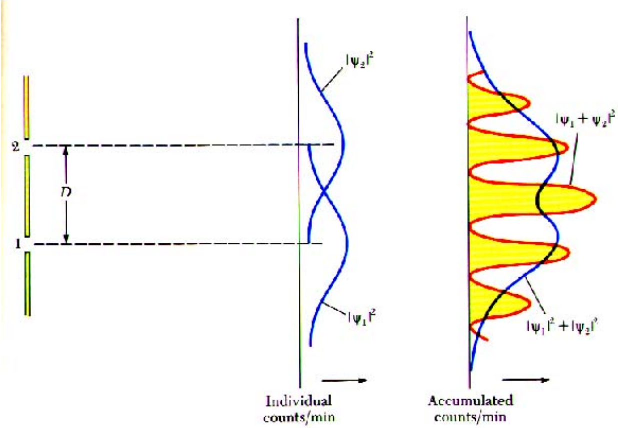


Figure: Interference pattern found in double-slit experiments.

Experiment 4: The double slit with monitoring

The monitoring proton destroys interference

Figure 6

Spin measurements

Wikipedia Commons

https://upload.wikimedia.org/wikipedia/commons/9/9e/Quantum_spin_and_the_Stern-Gerlach_experiment.ogv

- Elementary particles (such as electrons, photons, etc) and other atomic-scale systems have intrinsic properties called spin.
- Spin is genuinely quantum, i.e., there is no classical analogue.
- Its existence is inferred from experiments with an inhomogeneous magnetic field to deflect the particles (e.g. Stern-Gerlach).
- The magnetic field can be rotated 360 degrees, and the spin can be measured in any direction.
- Quantum spin assumes one of two values ('up', 'down'), classical angular momentum has continuous values.
- It usually suffices to measure spin in two orthogonal directions: up (U) - down (D) and left (L) - right (R)

Experiment 5: Spin

The Stern-Gerlach set-up

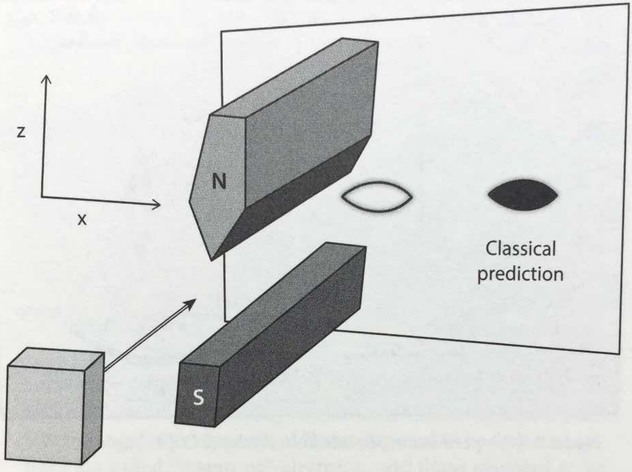


Figure 7

Experiment 5: Spin

Stern-Gerlach: confirmation of the quantization of spin

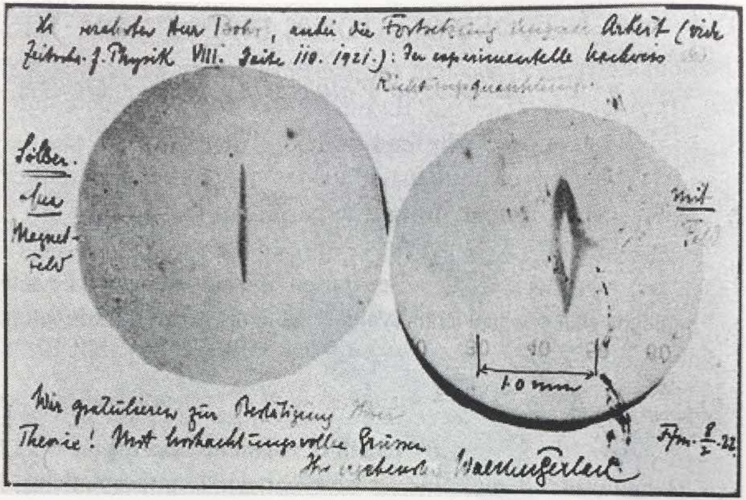
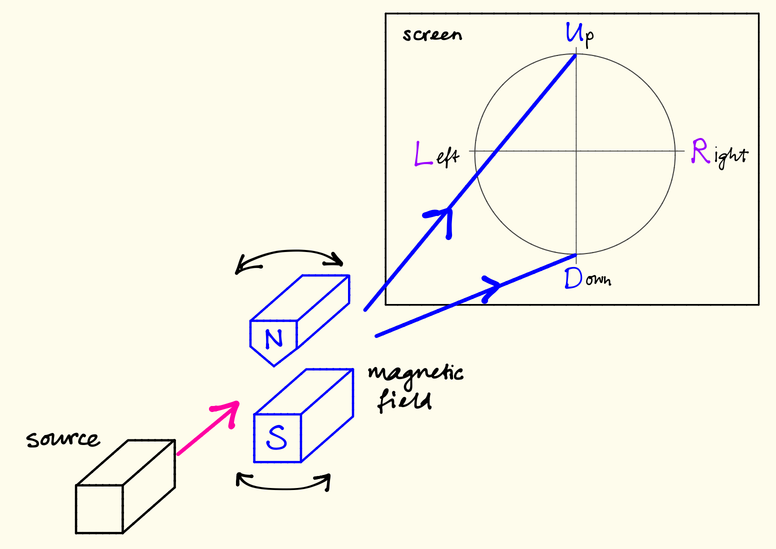
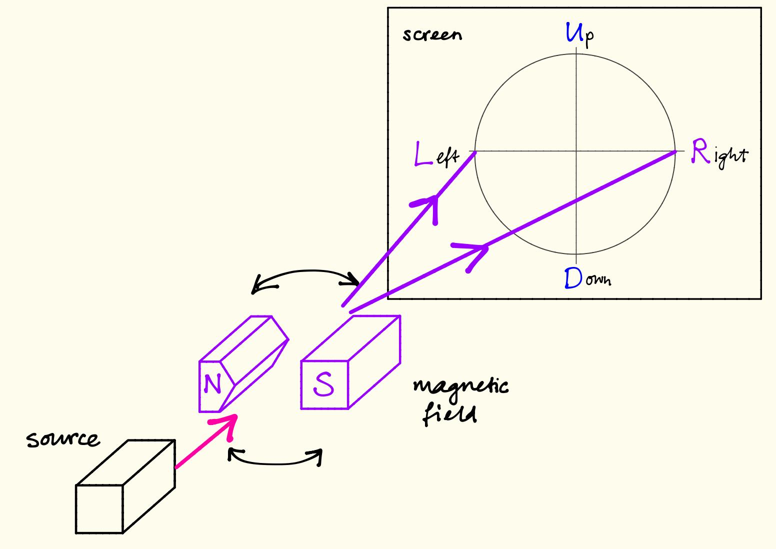


Figure 8. With permission of Niels Bohr Archive, Copenhagen.

Stern-Gerlach experiments: vertical



Stern-Gerlach experiments: horizontal



Combining Stern-Gerlach apparatuses

- The measurements are repeatable: if we let pass the same particle in two subsequent Stern-Gerlach apparatuses with the same orientation (and without tampering with the particle in the meantime), then the same measurement outcome will be observed (see Fig. 9(a)).

Question

Are the 'vertical' and the 'horizontal' spin properties related, i.e., are there correlations between the values for U-D and for L-R of the same particle?

- So let's combine Stern-Gerlach apparatuses (SG) with vertical (V) and horizontal orientation (H) (see Fig. 9(b)):

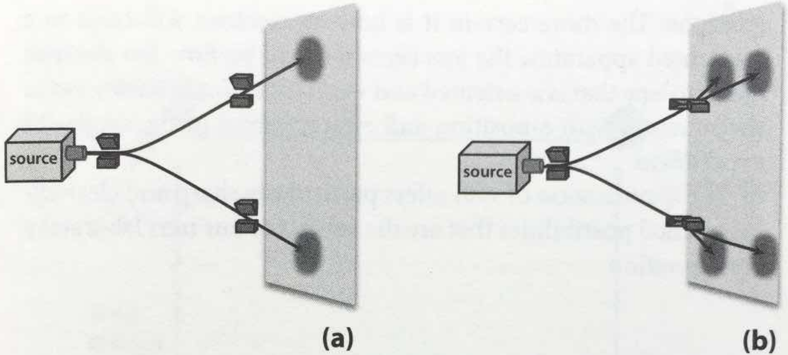


Figure 9

Spin measurements

- Exactly half of the particles coming out of the first SG will register with each outcome for the second SG.

⇒ So there appear to be no correlations, to have a spin in the vertical direction entails nothing about the spin in the horizontal direction and vice versa.

Three Stern-Gerlach experiments

- Suppose we have three SGs aligned for subsequent measurement, such that the first and third SG are oriented in the same direction as one another, but differently from the second: VHV or HVH
- No tampering with the particles between the measurements.
- Once the particles go into the third apparatus, they are presumably known to have a particular pair of spin properties (e.g. UL), so we should be able to predict the outcome of the third measurement.
- But it turns out that we cannot: precisely half the particles will register for either outcome...
- It seems as if the mere presence of the middle measurement constitutes some sort of tampering, as it randomizes the spin property of the particles in the other orientation.

Heisenberg's uncertainty principle

David Z Albert (1992). *Quantum Mechanics and Experience*. Harvard University Press.

- David Albert (1992, 7): it is impossible to assert that a particle has now such-and-such vertical spin and such-and-such horizontal spin
- This is an instance of Heisenberg's uncertainty principle, according to which some pairs of measurable properties ('observables') are incompatible in that the measurement of one of them disrupts the measurement of the other.

Spin measurements: summary

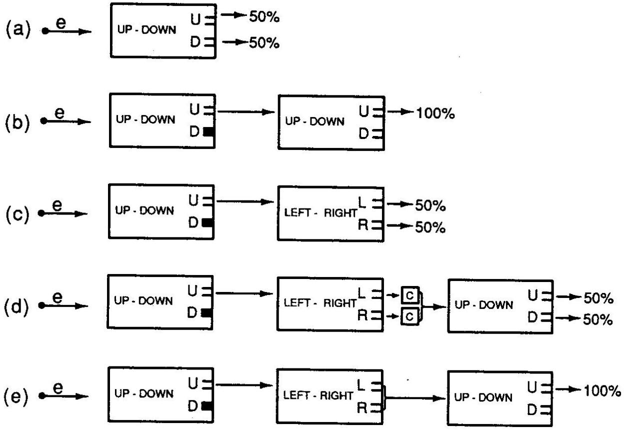
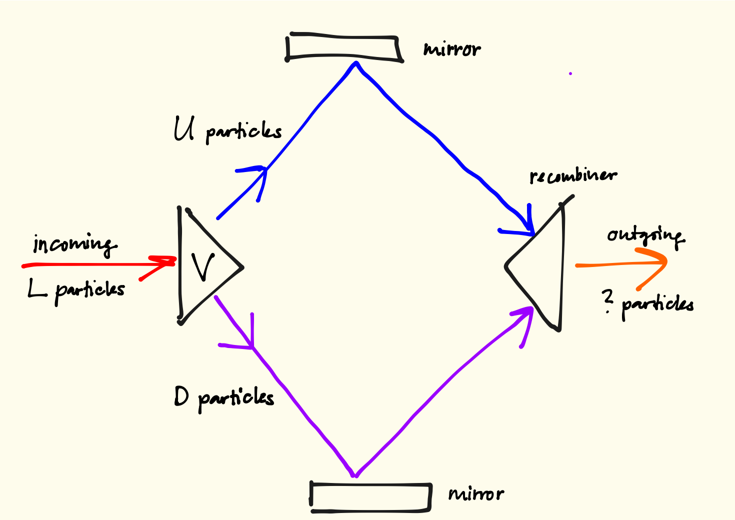


Figure: Stern-Gerlach experiment with 'mixture' (d) and 'superposition' (e) (Sklar, Fig. 4.4)

Experiment 6: The (Mach-Zehnder) interferometer

Entangling spin and position



- Suppose that L particles are fed into the device and we measure their V-spin after recombining. Expectation: we find half U and half D. And this is what we find.
- Suppose that U particles are fed into the device and we measure their H-spin after recombining. Expectation: we find half L and half R. And this is what we find.
- Suppose that L particles are fed into the device and we measure their H-spin after recombining. Expectation: we find half L and half R. But this is not at all what we find: all particles are found to be L!

- Add a sliding wall into the D-path. What happens if we slide the wall in?
- Expectation: overall output goes does down 50%; given that all particles were just found to be L, they should still be so, right?
- But they are not: only half of the particles are now L, the other half R.

Considering the paths of the particles: superposition

Which route does a particle take when the wall is out?

- Can it have taken the U-path? Apparently not, since these particles are known to randomize H-spin.
- Can it have taken the D-path? No, for the same reason.
- Can it somehow have taken both routes? Apparently not, since whenever we stop experiment and look to see where the particle is, we find it either on the U-path or on the D-path.
- Can it have taken neither route? No, since if we wall up both routes, nothing goes through.

Particles in superposition states

We can write these superposition states as follows (for later reference):

$$
\begin{aligned}
|\mathrm{L}\rangle &= \frac{1}{\sqrt{2}}|\mathrm{U}\rangle + \frac{1}{\sqrt{2}}|\mathrm{D}\rangle, \\
|\mathrm{R}\rangle &= \frac{1}{\sqrt{2}}|\mathrm{U}\rangle - \frac{1}{\sqrt{2}}|\mathrm{D}\rangle, \\
|\mathrm{U}\rangle &= \frac{1}{\sqrt{2}}|\mathrm{L}\rangle + \frac{1}{\sqrt{2}}|\mathrm{R}\rangle, \\
|\mathrm{D}\rangle &= \frac{1}{\sqrt{2}}|\mathrm{L}\rangle - \frac{1}{\sqrt{2}}|\mathrm{R}\rangle.
\end{aligned}
$$

Vectors and vector spaces

- We will use vectors—i.e., elements in a vector space—to represent physical states of quantum systems.
- Any two vectors in a vector space will yield another vector in the same space if added.

⇒ An 'addition' of any two possible states is another possible state.

⇒ 'superposition'

- Let's remind ourselves of some vector calculus, using the example of a two-dimensional vector space.
- The elements of such a space can be represented by arrows:

David Z Albert (1992). *Quantum Mechanics and Experience*. Harvard University Press.

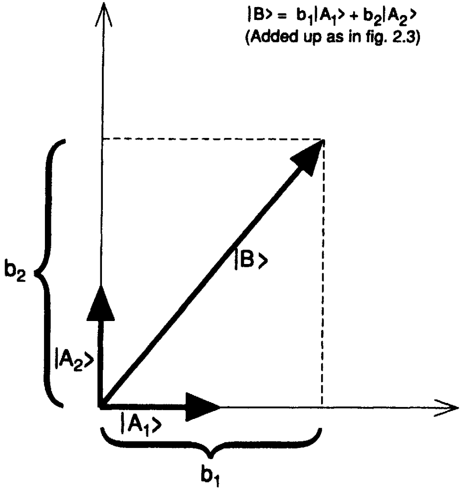


Figure: Figure 2.5 in Albert (1992)

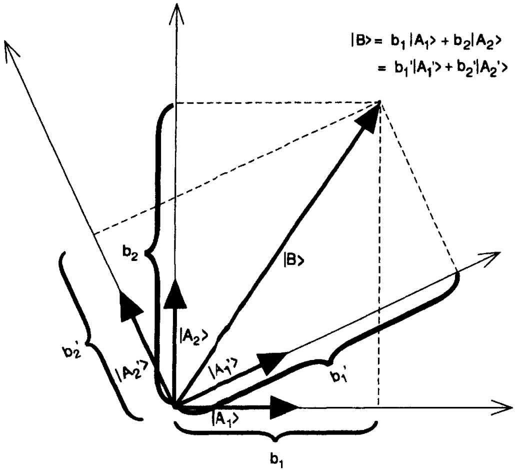


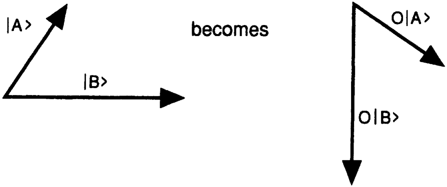
Figure: Figure 2.6 in Albert (1992)

Operators

Definition (Operator)

An operator $\hat{O}$ defined on a vector space V is a map or a prescription for taking every vector in V into some other vector in V.

Some examples: unit operator $\hat{1}$ transforming every vector into itself; 'multiply every vector by the number 7'; 'rotate every vector clockwise by 90° about some particular $|C\rangle$' (depicted below for a $|C\rangle$ pointing out of the page); 'map every vector into some particular $|A\rangle$'; etc.



Eigenvectors and eigenvalues

Definition (Eigenvectors and eigenvalues)

In case that for some particular operator $\hat{O}$ and some particular vector $|X\rangle$

$$\hat{O}|X\rangle = x|X\rangle$$

for some number x, then $|X\rangle$ is an eigenvector of $\hat{O}$, with eigenvalue x.

- certain vectors will in general be eigenvectors of some operators and not of others
- certain operators will in general have some vectors, and not others, as eigenvectors
- other operators will have other vectors as eigenvectors.
- The operator-eigenvector relation depends only on the vector and the operator in question, but not on the basis chosen.

Quantum mechanics: the five basic principles

Principle (A: Physical states)

Every physical system, composite or simple, is associated with some particular vector space. Every unit vector in this space (the 'state vectors') represents a possible physical state of the system. The states picked out by these vectors are taken to comprise all of the physically possible situations, although the correspondence is not one-to-one.

Principle (B: Observables)

Measurable properties of physical systems ('observables') are represented by (linear) operators on the vector spaces associated with those systems. The rule connecting the operators and the vectors states that if the vector happens to be an eigenvector (with eigenvalue, say, a) of an operator in question, then the state corresponding to the vector has the value a of that particular measurable property associated with the operator.

Back to the spin example

Construct a vector space in which the states U and D can be represented:

$$|U\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |D\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{1}$$

which implies $\langle U|D\rangle = 1 \cdot 0 + 0 \cdot 1 = 0$. In fact the two vectors in (1) form a basis of the vector space. Which operator should represent the observable 'V-spin'?

$$\text{V-spin operator} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{2}$$

where it is stipulated that eigenvalue $+1$ means 'U' and -1 'D' (vectors in (1) are eigenvectors of operator in (2)).

'L' and 'R' states are 'superpositions' of both the 'U' and the 'D' states, which means, since superposition states are states in the same vector space, that 'L' and 'R' ought to be representable in the same vector space. This plays out as follows:

$$|\mathrm{L}\rangle = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, \quad |\mathrm{R}\rangle = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}, \tag{3}$$

$$\text{H-spin operator} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

where 'H-spin $= +1$' means 'L' and 'H-spin $= -1$' means 'R'. Also, $\langle \mathrm{L}|\mathrm{R}\rangle = 0$, and $|\mathrm{L}\rangle$ and $|\mathrm{R}\rangle$ constitute another basis of the same space.

Recall that if $|A\rangle = (a_1, a_2)$ and $|B\rangle = (b_1, b_2)$ then

$$|A\rangle + |B\rangle = \begin{bmatrix} (a_1 + b_1) \\ (a_2 + b_2) \end{bmatrix}.$$

One can then see from (1), (2), and (3) how superposition and incompatibility is encoded in the formalism:

$$\begin{aligned} |\mathrm{L}\rangle &= \frac{1}{\sqrt{2}}|\mathrm{U}\rangle + \frac{1}{\sqrt{2}}|\mathrm{D}\rangle, \\ |\mathrm{R}\rangle &= \frac{1}{\sqrt{2}}|\mathrm{U}\rangle - \frac{1}{\sqrt{2}}|\mathrm{D}\rangle, \\ |\mathrm{U}\rangle &= \frac{1}{\sqrt{2}}|\mathrm{L}\rangle + \frac{1}{\sqrt{2}}|\mathrm{R}\rangle, \\ |\mathrm{D}\rangle &= \frac{1}{\sqrt{2}}|\mathrm{L}\rangle - \frac{1}{\sqrt{2}}|\mathrm{R}\rangle. \end{aligned} \tag{4}$$

Exercise

Convince yourself that $|U\rangle$ and $|D\rangle$ are not eigenvectors of the H-spin operator (and vice versa, mutatis mutandis).

Incompatibility of V-spin and H-spin

The V-spin and H-spin operators are incompatible with one another, in the sense that states of definite V-spin have no assignable H-spin value, and vice versa.

Principle (C: Dynamics)

Given the state of any physical system at any 'initial' time, and given the forces and constraints to which the system is subject, the Schrödinger equation gives a prescription whereby the state of that system at any other time is uniquely determined. This dynamics of the state vector is thus deterministic.

The dynamical laws are linear: if any state $|A\rangle$ at t_1 is evolved into another state $|A'\rangle$ at t_2 and any $|B\rangle$ at t_1 is evolved into $|B'\rangle$ at t_2, then $\alpha|A\rangle + \beta|B\rangle$ at t_1 is evolved into $\alpha|A'\rangle + \beta|B'\rangle$ at t_2.

We know what happens if we measure a state with respect to a particular property when that state is in an eigenstate of the operator corresponding to the measurable property in question (what?). But what happens if it isn't?

Principle (D: Connection with experiment, 'Born's rule')

A measurement of the observable $\hat{A}$ *is performed on a system in state* $|b\rangle$*, where the eigenvectors of* $\hat{A}$ *are* $|a_i\rangle$ *with eigenvalues* a_i*, i.e.* $\hat{A}|a_i\rangle = a_i|a_i\rangle$ *for all* i*. The probability that the outcome of such a measurement will be* a_i *is equal to*

$$|\langle b|a_i\rangle|^2.$$

Remarks:

- The number $|\langle b|a_i\rangle|^2$ is in the interval $[0, 1]$.
- In the special case when the system is in an eigenstate of the operator corresponding to the measurement, we get with probability 1 that the outcome is the eigenvalue associated with the eigenstate.
- Probability that L-particle is found to be U is 1/2, as it should be.

Principle (E: Collapse)

Whatever the state vector of a system S was just prior to a measurement of an observable O, the state vector of S just after the measurement must be an eigenvector of O with an eigenvalue corresponding to the outcome of that measurement.

Remarks:

- Which eigenvector the system jump into is determined by the outcome of the measurement; and this outcome, by Principle D, is a matter of probability.

$\Rightarrow$ element of chance, indeterminism, enters into the evolution of the state vector

Notice that Principle C was supposed to be a completely general account of how the state vector evolves under any circumstances, while Principle E seems to be a special case of C, but can't obviously be deduced it...

- Measurement in QM is (according to the standard view) a very active process that in general changes the measured system.

Albert (1992,38)

That's what's at the heart of the standard view. The rest... is details.