## First Three Replacement Rules

Conditional Exchange (CE)

$$\varphi \supset \psi \;::\; {\sim}\varphi \lor \psi$$

## First Three Replacement Rules

Double Negation (DN)

$$\varphi \;::\; {\sim}{\sim}\varphi$$

## First Three Replacement Rules

Commutation (comm)

$$\varphi \lor \psi \;::\; \psi \lor \varphi$$

$$\varphi \bullet \psi \;::\; \psi \bullet \varphi$$

## Differences between Inference Rules and replacement Rules

Can be used on part of a line

x. $A \supset (B \supset C)$
y. $A \supset ({\sim}B \lor C)$ x CE
z. ${\sim}A \lor ({\sim}B \lor C)$ y CE

## Differences between Inference Rules and replacement Rules

Can be used on part of a line

x. $A \supset (B \supset C)$
y. $A \supset ({\sim}B \lor C)$ x CE
z. ${\sim}A \lor ({\sim}B \lor C)$ y CE

## Differences between Inference Rules and replacement Rules

Can be used on part of a line

x. $A \supset (B \supset C)$
y. $A \supset ({\sim}B \lor C)$ x CE
z. ${\sim}A \lor ({\sim}B \lor C)$ y CE

## Differences between Inference Rules and replacement Rules

Can go in either direction

x. ~A ∨ (~B ∨ C)
y. A ⊃ (~B ∨ C)          x CE
z. A ⊃ (B ⊃ C)          y CE

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1. C v (D • E)
2. A ⊃ ~C
3. {~~A ⊃ (E • D)} ⊃ R          / ∴ ~~R

---

1. C v (D • E)
2. A ⊃ ~C
3. {~~A ⊃ (E • D)} ⊃ R          / ∴ ~~R
4. ~C ⊃ (D • E)          1 CE

---

1. C v (D • E)
2. A ⊃ ~C
3. {~~A ⊃ (E • D)} ⊃ R          / ∴ ~~R
4. ~C ⊃ (D • E)          1 CE
5. A ⊃ (D • E)          2, 4 HS

---

1. C v (D • E)
2. A ⊃ ~C
3. {~~A ⊃ (E • D)} ⊃ R          / ∴ ~~R
4. ~C ⊃ (D • E)          1 CE
5. A ⊃ (D • E)          2, 4 HS
6. ~~A ⊃ (D • E)          5 DN

---

1. C v (D • E)
2. A ⊃ ~C
3. {~~A ⊃ (E • D)} ⊃ R          / ∴ ~~R
4. ~C ⊃ (D • E)          1 CE
5. A ⊃ (D • E)          2, 4 HS
6. ~~A ⊃ (D • E)          5 DN
7. ~~A ⊃ (E • D)          6 comm

1. C v (D • E)
2. A ⊃ ~C
3. {~~A ⊃ (E • D)} ⊃ R / ∴ ~~R
4. ~C ⊃ (D • E) 1 CE
5. A ⊃ (D • E) 2, 4 HS
6. ~~A ⊃ (D • E) 5 DN
7. ~~A ⊃ (E • D) 6 comm
8. R 3, 7 MP

1. C v (D • E)
2. A ⊃ ~C
3. {~~A ⊃ (E • D)} ⊃ R / ∴ ~~R
4. ~C ⊃ (D • E) 1 CE
5. A ⊃ (D • E) 2, 4 HS
6. ~~A ⊃ (D • E) 5 DN
7. ~~A ⊃ (E • D) 6 comm
8. R 3, 7 MP
9. ~~R 8 DN

1. A v B
2. C v D
3. B ⊃ ~D / ∴ ~A ⊃ ~~C

1. A v B
2. C v D
3. B ⊃ ~D / ∴ ~A ⊃ ~~C
4. ~A ⊃ B 1 CE

1. A v B
2. C v D
3. B ⊃ ~D / ∴ ~A ⊃ ~~C
4. ~A ⊃ B 1 CE
5. D v C 2 comm

1. A v B
2. C v D
3. B ⊃ ~D / ∴ ~A ⊃ ~~C
4. ~A ⊃ B 1 CE
5. D v C 2 comm
6. ~D ⊃ C 5 CE

1. A v B
2. C v D
3. B ⊃ ~D / ∴ ~A ⊃ ~~C
4. ~A ⊃ B 1 CE
5. D v C 2 comm
6. ~D ⊃ C 5 CE
7. ~A ⊃ ~D 3, 4 HS

1. A v B
2. C v D
3. B ⊃ ~D / ∴ ~A ⊃ ~~C
4. ~A ⊃ B 1 CE
5. D v C 2 comm
6. ~D ⊃ C 5 CE
7. ~A ⊃ ~D 3, 4 HS
8. ~A ⊃ C 6, 7 HS

1. A v B
2. C v D
3. B ⊃ ~D / ∴ ~A ⊃ ~~C
4. ~A ⊃ B 1 CE
5. D v C 2 comm
6. ~D ⊃ C 5 CE
7. ~A ⊃ ~D 3, 4 HS
8. ~A ⊃ C 6, 7 HS
9. ~A ⊃ ~~C 8 DN

1. (P ⊃ Q) ⊃ K
2. K ⊃ (P • Q) / ∴(~Q ⊃ ~P) ⊃ (Q • P)

1. (P ⊃ Q) ⊃ K
2. K ⊃ (P • Q) / ∴(~Q ⊃ ~P) ⊃ (Q • P)
3. (~P v Q) ⊃ K 1 CE

1. (P ⊃ Q) ⊃ K
2. K ⊃ (P • Q) / ∴(~Q ⊃ ~P) ⊃ (Q • P)
3. (~P v Q) ⊃ K 1 CE
4. (Q v ~P) ⊃ K 3 comm

1. (P ⊃ Q) ⊃ K
2. K ⊃ (P • Q)   / ∴(~Q ⊃ ~P) ⊃ (Q • P)
3. (~P v Q) ⊃ K   1 CE
4. (Q v ~P) ⊃ K   3 comm
5. (~Q ⊃ ~P) ⊃ K   4 CE

1. (P ⊃ Q) ⊃ K
2. K ⊃ (P • Q)   / ∴(~Q ⊃ ~P) ⊃ (Q • P)
3. (~P v Q) ⊃ K   1 CE
4. (Q v ~P) ⊃ K   3 comm
5. (~Q ⊃ ~P) ⊃ K   4 CE
6. (~Q ⊃ ~P) ⊃ (P • Q)   5, 2 HS

1. (P ⊃ Q) ⊃ K
2. K ⊃ (P • Q)   / ∴(~Q ⊃ ~P) ⊃ (Q • P)
3. (~P v Q) ⊃ K   1 CE
4. (Q v ~P) ⊃ K   3 comm
5. (~Q ⊃ ~P) ⊃ K   4 CE
6. (~Q ⊃ ~P) ⊃ (P • Q)   5, 2 HS
7. (~Q ⊃ ~P) ⊃ (Q • P)   6 comm

## Last Seven Replacement Rules

DeMorgan's (DeM)

~(φ v ψ) :: ~φ • ~ψ

~(φ • ψ) :: ~φ v ~ψ

## Last Seven Replacement Rules

Biconditional Exchange (BE)

φ ≡ ψ :: (φ ⊃ ψ) • (ψ ⊃ φ)

## Last Seven Replacement Rules

Contraposition (contra)

φ ⊃ ψ :: ~ψ ⊃ ~φ

## Last Seven Replacement Rules

Distribution (dist)

φ • (ψ v χ) :: (φ • ψ) v (φ • χ)

φ v (ψ • χ) :: (φ v ψ) • (φ v χ)

## Last Seven Replacement Rules

Exportation (exp)

φ ⊃ (ψ ⊃ χ) :: (φ • ψ) ⊃ χ

## Last Seven Replacement Rules

Association (assoc)

(φ v ψ) v χ :: φ v (ψ v χ)

(φ • ψ) • χ :: φ • (ψ • χ)

## Last Seven Replacement Rules

Duplication (dup)

φ :: φ v φ

φ :: φ • φ

1. A / ∴ ~(~A • ~B) ⊃ ~P
2. ~(P ⊃ ~P) ⊃ (~A • ~B)

1. A / ∴ ~(~A • ~B) ⊃ ~P
2. ~(P ⊃ ~P) ⊃ (~A • ~B)
3. A v B 1 DI

1. A / ∴ ~(~A • ~B) ⊃ ~P
2. ~(P ⊃ ~P) ⊃ (~A • ~B)
3. A v B 1 DI
4. ~(~A • ~B) 3 DeM

1. A / ∴ ~(~A • ~B) ⊃ ~P
2. ~(P ⊃ ~P) ⊃ (~A • ~B)
3. A v B 1 DI
4. ~(~A • ~B) 3 DeM

~(φ • ψ) :: ~φ v ~ψ

1. A / ∴ ~(~A • ~B) ⊃ ~P
2. ~(P ⊃ ~P) ⊃ (~A • ~B)
3. A v B 1 DI
4. ~(~A • ~B) 3 DeM
5. ~(~A • ~B) ⊃ (P ⊃ ~P) 2 contra

1. A / ∴ ~(~A • ~B) ⊃ ~P
2. ~(P ⊃ ~P) ⊃ (~A • ~B)
3. A v B 1 DI
4. ~(~A • ~B) 3 DeM
5. ~(~A • ~B) ⊃ (P ⊃ ~P) 2 contra
6. ~(~A • ~B) ⊃ (~P v ~P) 5 CE

1. A / ∴ ~(~A • ~B) ⊃ ~P
2. ~(P ⊃ ~P) ⊃ (~A • ~B)
3. A v B 1 DI
4. ~(~A • ~B) 3 DeM
5. ~(~A • ~B) ⊃ (P ⊃ ~P) 2 contra
6. ~(~A • ~B) ⊃ (~P v ~P) 5 CE
7. ~(~A • ~B) ⊃ ~P 6 dup

01. (I v S ) ⊃ C
02. D ⊃ (P • I) / ∴ D ⊃ C

01. (I v S ) ⊃ C
02. D ⊃ (P • I)　　　/ ∴ D ⊃ C

(I v S ) ⊃ C -> I ⊃ C

D ⊃ (P • I) -> D ⊃ I

01. (I v S ) ⊃ C
02. D ⊃ (P • I)　　　/ ∴ D ⊃ C

(I v S ) ⊃ C -> I ⊃ C

D ⊃ (P • I) -> D ⊃ I

01. (I v S ) ⊃ C
02. D ⊃ (P • I)　　　/ ∴ D ⊃ C
03. ~D v (P • I)　　　2 CE

01. (I v S ) ⊃ C
02. D ⊃ (P • I)　　　/ ∴ D ⊃ C
03. ~D v (P • I)　　　2 CE
04. (~D v P) • (~D v I)　　　3 dist

01. (I v S ) ⊃ C
02. D ⊃ (P • I)　　　/ ∴ D ⊃ C
03. ~D v (P • I)　　　2 CE
04. (~D v P) • (~D v I)　　　3 dist

$\varphi \vee (\psi \bullet \chi) :: (\varphi \vee \psi) \bullet (\varphi \vee \chi)$

01. (I v S ) ⊃ C
02. D ⊃ (P • I)　　　/ ∴ D ⊃ C
03. ~D v (P • I)　　　2 CE
04. (~D v P) • (~D v I)　　　3 dist
05. ~D v I　　　4 simp

01. (I v S ) ⊃ C
02. D ⊃ (P • I) / ∴ D ⊃ C
03. ~D v (P • I) 2 CE
04. (~D v P) • (~D v I) 3 dist
05. ~D v I 4 simp
06. D ⊃ I 5 CE

01. (I v S ) ⊃ C
02. D ⊃ (P • I) / ∴ D ⊃ C
03. ~D v (P • I) 2 CE
04. (~D v P) • (~D v I) 3 dist
05. ~D v I 4 simp
06. D ⊃ I 5 CE
07. ~C ⊃ ~(I v S) 1 contra

01. (I v S ) ⊃ C
02. D ⊃ (P • I) / ∴ D ⊃ C
03. ~D v (P • I) 2 CE
04. (~D v P) • (~D v I) 3 dist
05. ~D v I 4 simp
06. D ⊃ I 5 CE
07. ~C ⊃ ~(I v S) 1 contra
08. C v ~(I v S) **7 CE**

01. (I v S ) ⊃ C
02. D ⊃ (P • I) / ∴ D ⊃ C
03. ~D v (P • I) 2 CE
04. (~D v P) • (~D v I) 3 dist
05. ~D v I 4 simp
06. D ⊃ I 5 CE
07. ~C ⊃ ~(I v S) 1 contra
08. C v ~(I v S) **7 CE**
09. C v (~I • ~S) 8 DeM

01. (I v S ) ⊃ C
02. D ⊃ (P • I) / ∴ D ⊃ C
03. ~D v (P • I) 2 CE
04. (~D v P) • (~D v I) 3 dist
05. ~D v I 4 simp
06. D ⊃ I 5 CE
07. ~C ⊃ ~(I v S) 1 contra
08. C v ~(I v S) **7 CE**
09. C v (~I • ~S) 8 DeM
10. (C v ~I) • (C v ~S) 9 dist

01. (I v S ) ⊃ C
02. D ⊃ (P • I) / ∴ D ⊃ C
03. ~D v (P • I) 2 CE
04. (~D v P) • (~D v I) 3 dist
05. ~D v I 4 simp
06. D ⊃ I 5 CE
07. ~C ⊃ ~(I v S) 1 contra
08. C v ~(I v S) **7 CE**
09. C v (~I • ~S) 8 DeM
10. (C v ~I) • (C v ~S) 9 dist
11. C v ~I 10 simp

| | | |
|---|---|---|
| 01. | (I v S ) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | ~D v (P • I) | 2 CE |
| 04. | (~D v P) • (~D v I) | 3 dist |
| 05. | ~D v I | 4 simp |
| 06. | D ⊃ I | 5 CE |
| 07. | ~C ⊃ ~(I v S) | 1 contra |
| 08. | C v ~(I v S) | **7 CE** |
| 09. | C v (~I • ~S) | 8 DeM |
| 10. | (C v ~I) • (C v ~S) | 9 dist |
| 11. | C v ~I | 10 simp |
| 12. | ~I v C | 11 comm |

| | | |
|---|---|---|
| 01. | (I v S ) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | ~D v (P • I) | 2 CE |
| 04. | (~D v P) • (~D v I) | 3 dist |
| 05. | ~D v I | 4 simp |
| 06. | D ⊃ I | 5 CE |
| 07. | ~C ⊃ ~(I v S) | 1 contra |
| 08. | C v ~(I v S) | **7 CE** |
| 09. | C v (~I • ~S) | 8 DeM |
| 10. | (C v ~I) • (C v ~S) | 9 dist |
| 11. | C v ~I | 10 simp |
| 12. | ~I v C | 11 comm |
| 13. | I ⊃ C | 12 CE |

| | | |
|---|---|---|
| 01. | (I v S ) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | ~D v (P • I) | 2 CE |
| 04. | (~D v P) • (~D v I) | 3 dist |
| 05. | ~D v I | 4 simp |
| 06. | D ⊃ I | 5 CE |
| 07. | ~C ⊃ ~(I v S) | 1 contra |
| 08. | C v ~(I v S) | **7 CE** |
| 09. | C v (~I • ~S) | 8 DeM |
| 10. | (C v ~I) • (C v ~S) | 9 dist |
| 11. | C v ~I | 10 simp |
| 12. | ~I v C | 11 comm |
| 13. | I ⊃ C | 12 CE |
| 14. | D ⊃ C | 6, 13 HS |

## Indirect Proof

One of two 'proof strategies', the other is conditional proof

Basic idea: prove that φ is true, not by deriving φ directly, but by showing that if the premises are all true, ~φ must be false.

## Indirect Proof

At some point in a proof, you decide you'd like to be able to derive φ on a line, but you can't figure out how. Add an assumption line consisting of ~φ, then proceed using the rules.

## Indirect Proof

Keep deriving lines until you derive an explicit contradiction. We know that contradictions are always false. But we also know that our rules are truth preserving, and so if they are applied to only true statements they will produce only true statements.

## Indirect Proof

But: we managed to produce a false statement, the explicit contradiction. So, the set of statements we were applying the rules to must not all have been true.

## Indirect Proof

But the only statement that is suspect is the one we added as an assumption: $\sim\varphi$. So that one must be the one that is false. And if $\sim\varphi$ is false, then $\varphi$ must be true. So you are justified in writing a new derived line consisting of $\varphi$.

1. A ⊃ B
2. B ⊃ ~B / ∴ ~A

1. A ⊃ B
2. B ⊃ ~B / ∴ ~A
3. ~B v ~B 2 CE

1. A ⊃ B
2. B ⊃ ~B / ∴ ~A
3. ~B v ~B 2 CE
4. ~B 3 dup

1. A ⊃ B
2. B ⊃ ~B / ∴ ~A
3. ~B v ~B 2 CE
4. ~B 3 dup
5. ~A 4, 1 MT

1. A ⊃ B
2. B ⊃ ~B / ∴ ~A

---

1. A ⊃ B
2. B ⊃ ~B / ∴ ~A

3. A AIP

---

1. A ⊃ B
2. B ⊃ ~B / ∴ ~A

3. ⟶ A AIP

---

1. A ⊃ B
2. B ⊃ ~B / ∴ ~A

3. ⟶ A AIP
4. B 1, 3 MP

---

1. A ⊃ B
2. B ⊃ ~B / ∴ ~A

3. ⟶ A AIP
4. B 1, 3 MP
5. **~B** 4, 2 MP

---

1. A ⊃ B
2. B ⊃ ~B / ∴ ~A

3. ⟶ A AIP
4. B 1, 3 MP
5. **~B** 4, 2 MP
6. **B • ~B** 4, 5 conj

| | | |
|---|---|---|
| 1. | A ⊃ B | |
| 2. | B ⊃ ~B | / ∴ ~A |
| 3. | → A | AIP |
| 4. | B | 1, 3 MP |
| 5. | **~B** | 4, 2 MP |
| 6. | **B • ~B** | 4, 5 conj |

| | | |
|---|---|---|
| 1. | A ⊃ B | |
| 2. | B ⊃ ~B | / ∴ ~A |
| 3. | → A | AIP |
| 4. | B | 1, 3 MP |
| 5. | **~B** | 4, 2 MP |
| 6. | **B • ~B** | 4, 5 conj |
| 7. | **~A** | 3-6 IP |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | → ~M | AIP |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | → ~M | AIP |
| 04. | ~M v P | 3 DI |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | → ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |
| 06. | K | 5 simp |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |
| 06. | K | 5 simp |
| 07. | ~L | 5 simp |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |
| 06. | K | 5 simp |
| 07. | ~L | 5 simp |
| 08. | ~K | 7, 2 DS |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |
| 06. | K | 5 simp |
| 07. | ~L | 5 simp |
| 08. | ~K | 7, 2 DS |
| 09. | K • ~K | 6, 8 conj |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |
| 06. | K | 5 simp |
| 07. | ~L | 5 simp |
| 08. | ~K | 7, 2 DS |
| 09. | K • ~K | 6, 8 conj |
| 10. | M | 3-9 IP |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |
| 06. | K | 5 simp |
| 07. | ~L | 5 simp |
| 08. | ~K | 7, 2 DS |
| 09. | K • ~K | 6, 8 conj |
| 10. | M | 3-9 IP |
| 11. | M v K | 10 DI |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | ⟶~(D ⊃ C) | AIP |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | ⟶~(D ⊃ C) | AIP |
| 04. | ~(~D v C) | 3 CE |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | ⟶~(D ⊃ C) | AIP |
| 04. | ~(~D v C) | 3 CE |
| 05. | D • ~C | 4 DeM |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | ⟶~(D ⊃ C) | AIP |
| 04. | ~(~D v C) | 3 CE |
| 05. | D • ~C | 4 DeM |
| 06. | D | 5 simp |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | ⟶~(D ⊃ C) | AIP |
| 04. | ~(~D v C) | 3 CE |
| 05. | D • ~C | 4 DeM |
| 06. | D | 5 simp |
| 07. | ~C | 5 simp |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | → ~(D ⊃ C) | AIP |
| 04. | ~(~D v C) | 3 CE |
| 05. | D • ~C | 4 DeM |
| 06. | D | 5 simp |
| 07. | ~C | 5 simp |
| 08. | P • I | 6, 2 MP |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | → ~(D ⊃ C) | AIP |
| 04. | ~(~D v C) | 3 CE |
| 05. | D • ~C | 4 DeM |
| 06. | D | 5 simp |
| 07. | ~C | 5 simp |
| 08. | P • I | 6, 2 MP |
| 09. | I | 8 simp |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | → ~(D ⊃ C) | AIP |
| 04. | ~(~D v C) | 3 CE |
| 05. | D • ~C | 4 DeM |
| 06. | D | 5 simp |
| 07. | ~C | 5 simp |
| 08. | P • I | 6, 2 MP |
| 09. | I | 8 simp |
| 10. | I v S | 9 DI |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | → ~(D ⊃ C) | AIP |
| 04. | ~(~D v C) | 3 CE |
| 05. | D • ~C | 4 DeM |
| 06. | D | 5 simp |
| 07. | ~C | 5 simp |
| 08. | P • I | 6, 2 MP |
| 09. | I | 8 simp |
| 10. | I v S | 9 DI |
| 11. | C | 1, 10 MP |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | → ~(D ⊃ C) | AIP |
| 04. | ~(~D v C) | 3 CE |
| 05. | D • ~C | 4 DeM |
| 06. | D | 5 simp |
| 07. | ~C | 5 simp |
| 08. | P • I | 6, 2 MP |
| 09. | I | 8 simp |
| 10. | I v S | 9 DI |
| 11. | C | 1, 10 MP |
| 12. | C • ~C | 7, 11 conj |

| | | |
|---|---|---|
| 01. | (I v S) ⊃ C | |
| 02. | D ⊃ (P • I) | / ∴ D ⊃ C |
| 03. | → ~(D ⊃ C) | AIP |
| 04. | ~(~D v C) | 3 CE |
| 05. | D • ~C | 4 DeM |
| 06. | D | 5 simp |
| 07. | ~C | 5 simp |
| 08. | P • I | 6, 2 MP |
| 09. | I | 8 simp |
| 10. | I v S | 9 DI |
| 11. | C | 1, 10 MP |
| 12. | C • ~C | 7, 11 conj |
| 13. | D ⊃ C | 3-12 IP |

## Rules for the use of IP

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1. Start subproof (SP) by indenting and designating first line AIP

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1. Start subproof (SP) by indenting and designating first line AIP
2. IP ends only when an explicit contradiction is derived
3. Mark off IP, closing SP and discharging assumption
4. The next line after the closed IP SP can only be the negation of the AIP

## Rules for the use of IP

5. All SPs must be closed before the proof can end

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | → ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |
| 06. | K | 5 simp |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | → ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |
| 06. | K | 5 simp |
| 07. | M v K | 6 DI |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | → ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |
| 06. | K | 5 simp |
| 07. | M v K | 6 DI |

Rules for the use of IP

5. All SPs must be closed before the proof can end
6. Once a SP has been closed, no lines in it may be used or cited

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | → ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |
| 06. | K | 5 simp |
| 07. | ~L | 5 simp |
| 08. | ~K | 7, 2 DS |
| 09. | K • ~K | 6, 8 conj |
| 10. | M | 3-9 IP |

| | | |
|---|---|---|
| 01. | (~M v P) ⊃ (K • ~L) | |
| 02. | ~K v L | / ∴ M v K |
| 03. | → ~M | AIP |
| 04. | ~M v P | 3 DI |
| 05. | K • ~L | 1, 4 MP |
| 06. | K | 5 simp |
| 07. | ~L | 5 simp |
| 08. | ~K | 7, 2 DS |
| 09. | K • ~K | 6, 8 conj |
| 10. | M | 3-9 IP |
| 11. | M v K | 6 DI |

01. (~M v P) ⊃ (K • ~L)
02. ~K v L / ∴ M v K
03. → ~M AIP
04. ~M v P 3 DI
05. K • ~L 1, 4 MP
06. K 5 simp
07. ~L 5 simp
08. ~K 7, 2 DS
09. K • ~K 6, 8 conj
10. M 3-9 IP
11. M v K 6 DI

## Tautologies

The proof method is a method for demonstrating validity, for demonstrating that if a given set of statements (premises) is true, then another statement (the conclusion) is true. Given this, is there any way we could use the proof method to show that a statement is a tautology?

## Tautologies

Yes. If a statement can be derived from no premises, then we know that that statement follows from anything, it will follow from any set of premises. (If you can derive it from no premises, then clearly if you had premises, regardless of what those premises were, you would also be able to derive it.)

## Tautologies

This means that any argument that has this statement as its conclusion is valid. And if this is true, then the statement must be a tautology. No statement of any other category could possibly have this property.

/ ∴ (P ⊃ ~P) ⊃ ~P

/ ∴ (P ⊃ ~P) ⊃ ~P
1. → P ⊃ ~P ACP

/ ∴ (P ⊃ ~P) ⊃ ~P

| | | |
|---|---|---|
| 1. | → P ⊃ ~P | ACP |
| 2. | ~P v ~P | 1 CE |

/ ∴ (P ⊃ ~P) ⊃ ~P

| | | |
|---|---|---|
| 1. | → P ⊃ ~P | ACP |
| 2. | ~P v ~P | 1 CE |
| 3. | ~P | 2 dup |

/ ∴ (P ⊃ ~P) ⊃ ~P

| | | |
|---|---|---|
| 1. | → P ⊃ ~P | ACP |
| 2. | ~P v ~P | 1 CE |
| 3. | ~P | 2 dup |

/ ∴ (P ⊃ ~P) ⊃ ~P

| | | |
|---|---|---|
| 1. | → P ⊃ ~P | ACP |
| 2. | ~P v ~P | 1 CE |
| 3. | ~P | 2 dup |
| 4. | (P ⊃ ~P) ⊃ ~P | 1-3 CP |

/ ∴ ~P ⊃ (P ⊃ ~P)

/ ∴ ~P ⊃ (P ⊃ ~P)

| | | |
|---|---|---|
| 1. | → ~P | ACP |

/ ∴ ~P ⊃ (P ⊃ ~P)

| | | |
|---|---|---|
| 1. | → ~P | ACP |
| 2. | ~P v ~P | 1 dup |

/ ∴ ~P ⊃ (P ⊃ ~P)

| | | |
|---|---|---|
| 1. | → ~P | ACP |
| 2. | ~P v ~P | 1 dup |
| 3. | P ⊃ ~P | 2 CE |

/ ∴ ~P ⊃ (P ⊃ ~P)

| | | |
|---|---|---|
| 1. | → ~P | ACP |
| 2. | ~P v ~P | 1 dup |
| 3. | P ⊃ ~P | 2 CE |

/ ∴ ~P ⊃ (P ⊃ ~P)

| | | |
|---|---|---|
| 1. | → ~P | ACP |
| 2. | ~P v ~P | 1 dup |
| 3. | P ⊃ ~P | 2 CE |
| 4. | ~P ⊃ (P ⊃ ~P) | 1-3 CP |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | → Q | ACP |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | → Q | ACP |
| 02. | Q • Q | 1 dup |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | → Q | ACP |
| 02. | Q • Q | 1 dup |
| 03. | Q | 2 simp |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | → Q | ACP |
| 02. | Q • Q | 1 dup |
| 03. | Q | 2 simp |
| 04. | Q ⊃ Q | 1-3 CP |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | → Q | ACP |
| 02. | Q • Q | 1 dup |
| 03. | Q | 2 simp |
| 04. | Q ⊃ Q | 1-3 CP |
| 05. | → (P ⊃ Q) • (Q ⊃ R) | ACP |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | → Q | ACP |
| 02. | Q • Q | 1 dup |
| 03. | Q | 2 simp |
| 04. | Q ⊃ Q | 1-3 CP |
| 05. | → (P ⊃ Q) • (Q ⊃ R) | ACP |
| 06. | P ⊃ Q | 5 simp |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | → Q | ACP |
| 02. | Q • Q | 1 dup |
| 03. | Q | 2 simp |
| 04. | Q ⊃ Q | 1-3 CP |
| 05. | → (P ⊃ Q) • (Q ⊃ R) | ACP |
| 06. | P ⊃ Q | 5 simp |
| 07. | Q ⊃ R | 5 simp |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | Q | ACP |
| 02. | Q • Q | 1 dup |
| 03. | Q | 2 simp |
| 04. | Q ⊃ Q | 1-3 CP |
| 05. | (P ⊃ Q) • (Q ⊃ R) | ACP |
| 06. | P ⊃ Q | 5 simp |
| 07. | Q ⊃ R | 5 simp |
| 08. | ~Q ⊃ ~P | 6 contra |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | Q | ACP |
| 02. | Q • Q | 1 dup |
| 03. | Q | 2 simp |
| 04. | Q ⊃ Q | 1-3 CP |
| 05. | (P ⊃ Q) • (Q ⊃ R) | ACP |
| 06. | P ⊃ Q | 5 simp |
| 07. | Q ⊃ R | 5 simp |
| 08. | ~Q ⊃ ~P | 6 contra |
| 09. | ~Q v Q | 4 CE |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | Q | ACP |
| 02. | Q • Q | 1 dup |
| 03. | Q | 2 simp |
| 04. | Q ⊃ Q | 1-3 CP |
| 05. | (P ⊃ Q) • (Q ⊃ R) | ACP |
| 06. | P ⊃ Q | 5 simp |
| 07. | Q ⊃ R | 5 simp |
| 08. | ~Q ⊃ ~P | 6 contra |
| 09. | ~Q v Q | 4 CE |
| 10. | ~P v R | 7, 8, 9 dil |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | Q | ACP |
| 02. | Q • Q | 1 dup |
| 03. | Q | 2 simp |
| 04. | Q ⊃ Q | 1-3 CP |
| 05. | (P ⊃ Q) • (Q ⊃ R) | ACP |
| 06. | P ⊃ Q | 5 simp |
| 07. | Q ⊃ R | 5 simp |
| 08. | ~Q ⊃ ~P | 6 contra |
| 09. | ~Q v Q | 4 CE |
| 10. | ~P v R | 7, 8, 9 dil |
| 11. | P ⊃ R | 10 CE |

/ ∴ {(P ⊃ Q) • (Q ⊃ R)} ⊃ (P ⊃ R)

| | | |
|---|---|---|
| 01. | Q | ACP |
| 02. | Q • Q | 1 dup |
| 03. | Q | 2 simp |
| 04. | Q ⊃ Q | 1-3 CP |
| 05. | (P ⊃ Q) • (Q ⊃ R) | ACP |
| 06. | P ⊃ Q | 5 simp |
| 07. | Q ⊃ R | 5 simp |
| 08. | ~Q ⊃ ~P | 6 contra |
| 09. | ~Q v Q | 4 CE |
| 10. | ~P v R | 7, 8, 9 dil |
| 11. | P ⊃ R | 10 CE |
| 12. | {(P⊃Q)•(Q⊃R)}⊃(P⊃R) | 5-11 CP |

01. F / ∴ (G ⊃ H) v (~G ⊃ J)

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. ⟶~(G ⊃ H) ACP

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. ⟶~(G ⊃ H) ACP
03. ~(~G v H) 2 CE

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. ⟶~(G ⊃ H) ACP
03. ~(~G v H) 2 CE
04. G • ~H 3 DeM

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. ⟶~(G ⊃ H) ACP
03. ~(~G v H) 2 CE
04. G • ~H 3 DeM
05. G 4 simp

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. ⟶~(G ⊃ H) ACP
03. ~(~G v H) 2 CE
04. G • ~H 3 DeM
05. G 4 simp
06. G v J 5 DI

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. ⟶~(G ⊃ H) ACP
03. ~(~G v H) 2 CE
04. G • ~H 3 DeM
05. G 4 simp
06. G v J 5 DI
07. ~G ⊃ J 6 CE

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. → ~(G ⊃ H) ACP
03. ~(~G v H) 2 CE
04. G • ~H 3 DeM
05. G 4 simp
06. G v J 5 DI
07. ~G ⊃ J 6 CE

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. → ~(G ⊃ H) ACP
03. ~(~G v H) 2 CE
04. G • ~H 3 DeM
05. G 4 simp
06. G v J 5 DI
07. ~G ⊃ J 6 CE
08. ~(G ⊃ H) ⊃ (~G ⊃ J) 2-7 CP

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. → ~(G ⊃ H) ACP
03. ~(~G v H) 2 CE
04. G • ~H 3 DeM
05. G 4 simp
06. G v J 5 DI
07. ~G ⊃ J 6 CE
08. ~(G ⊃ H) ⊃ (~G ⊃ J) 2-7 CP
09. (G ⊃ H) v (~G ⊃ J) 8 CE

01. F / ∴ (G ⊃ H) v (~G ⊃ J)

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. → G ACP

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. → G ACP
03. G • G 2 dup

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. → G ACP
03. G • G 2 dup
04. G 3 simp

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. → G ACP
03. G • G 2 dup
04. G 3 simp

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. → G ACP
03. G • G 2 dup
04. G 3 simp
05. G ⊃ G 2-4 CP

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. → G ACP
03. G • G 2 dup
04. G 3 simp
05. G ⊃ G 2-4 CP
06. ~G v G 5 CE

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. → G ACP
03. G • G 2 dup
04. G 3 simp
05. G ⊃ G 2-4 CP
06. ~G v G 5 CE
07. (~G v G) v (H v J) 6 DI

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. → G ACP
03. G • G 2 dup
04. G 3 simp
05. G ⊃ G 2-4 CP
06. ~G v G 5 CE
07. (~G v G) v (H v J) 6 DI
08. {(~G v G) v H} v J 7 assoc

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. G ACP
03. G • G 2 dup
04. G 3 simp
05. G ⊃ G 2-4 CP
06. ~G v G 5 CE
07. (~G v G) v (H v J) 6 DI
08. {(~G v G) v H} v J 7 assoc
09. {~G v (G v H)} v J 8 assoc

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. G ACP
03. G • G 2 dup
04. G 3 simp
05. G ⊃ G 2-4 CP
06. ~G v G 5 CE
07. (~G v G) v (H v J) 6 DI
08. {(~G v G) v H} v J 7 assoc
09. {~G v (G v H)} v J 8 assoc
10. {~G v (H v G)} v J 9 comm

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. G ACP
03. G • G 2 dup
04. G 3 simp
05. G ⊃ G 2-4 CP
06. ~G v G 5 CE
07. (~G v G) v (H v J) 6 DI
08. {(~G v G) v H} v J 7 assoc
09. {~G v (G v H)} v J 8 assoc
10. {~G v (H v G)} v J 9 comm
11. {(~G v H) v G} v J 10 assoc

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. G ACP
03. G • G 2 dup
04. G 3 simp
05. G ⊃ G 2-4 CP
06. ~G v G 5 CE
07. (~G v G) v (H v J) 6 DI
08. {(~G v G) v H} v J 7 assoc
09. {~G v (G v H)} v J 8 assoc
10. {~G v (H v G)} v J 9 comm
11. {(~G v H) v G} v J 10 assoc
12. (~G v H) v (G v J) 11 assoc

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. G ACP
03. G • G 2 dup
04. G 3 simp
05. G ⊃ G 2-4 CP
06. ~G v G 5 CE
07. (~G v G) v (H v J) 6 DI
08. {(~G v G) v H} v J 7 assoc
09. {~G v (G v H)} v J 8 assoc
10. {~G v (H v G)} v J 9 comm
11. {(~G v H) v G} v J 10 assoc
12. (~G v H) v (G v J) 11 assoc
13. (~G v H) v (~G ⊃ J) 12 CE

01. F / ∴ (G ⊃ H) v (~G ⊃ J)
02. G ACP
03. G • G 2 dup
04. G 3 simp
05. G ⊃ G 2-4 CP
06. ~G v G 5 CE
07. (~G v G) v (H v J) 6 DI
08. {(~G v G) v H} v J 7 assoc
09. {~G v (G v H)} v J 8 assoc
10. {~G v (H v G)} v J 9 comm
11. {(~G v H) v G} v J 10 assoc
12. (~G v H) v (G v J) 11 assoc
13. (~G v H) v (~G ⊃ J) 12 CE
14. (G ⊃ H) v (~G ⊃ J) 13 CE