Proofs: Inference Rules

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1. Introduction to the proof method

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2. Two inference rules:
MP, MT, examples

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3. Discussion

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4. DS, HS, simp, examples

Introduction to the Proof Method

1. Representation of argument

Introduction to the Proof Method

1. Representation of argument

1. (A • B) ⊃ C

2. A • B / ∴ C

Introduction to the Proof Method

2. How does it work?

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E

Introduction to the Proof Method

2. How does it work?

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E

Use rules to derive new numbered lines from existing numbered lines

Introduction to the Proof Method

2. How does it work?

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E

Goal is to derive a numbered line that is the conclusion

Introduction to the Proof Method

2. How does it work?

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E
3. ~D 2 simp

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New numbered line

Introduction to the Proof Method

2. How does it work?

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E
3. ~D 2 simp

'Justification' : the rule and prior lines used to derive the new line

Introduction to the Proof Method

2. How does it work?

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E
3. ~D 2 simp
4. E ⊃ ~G 3, 1 DS

Introduction to the Proof Method

2. How does it work?

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E
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4. E ⊃ ~G 3, 1 DS
5. G 2 simp

Introduction to the Proof Method

2. How does it work?

1. D v (E ⊃ ~G)
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4. E ⊃ ~G 3, 1 DS
5. G 2 simp
6. ~E 5, 4 MT

Introduction to the Proof Method

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Introduction to the Proof Method

3. Why does it work?

Introduction to the Proof Method

3. Why does it work?

Rules are Truth Preserving

If they are applied to only true lines, they will produce only true lines

1. D v (E ⊃ ~G)	
2. ~D • G	/ ∴ ~E
3. ~D	2 simp
4. E ⊃ ~G	3, 1 DS
5. G	2 simp
6. ~E	5, 4 MT

If these are true, then that is true

1. D v (E ⊃ ~G)	
2. ~D • G	/ ∴ ~E
3. ~D	2 simp
4. E ⊃ ~G	3, 1 DS
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2. ~D • G	/ ∴ ~E
3. ~D	2 simp
4. E ⊃ ~G	3, 1 DS
5. G	2 simp
6. ~E	5, 4 MT

So if the premises are true, then all the derived lines must be true. And the conclusion is one of the derived lines.

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E
3. ~D 2 simp
4. E ⊃ ~G 3, 1 DS
5. G 2 simp
6. ~E 5, 4 MT

So if the premises are true, then the conclusion must be true. And this is the definition of validity.

First two Inference Rules

Modus Ponens (MP)

φ ⊃ ψ
φ
∴ ψ

First two Inference Rules

Modus Tollens (MT)

φ ⊃ ψ
~ψ
∴ ~φ

1. (A • B) ⊃ C
2. A • B / ∴ C

1. (A • B) ⊃ C
2. A • B / ∴ C
3. C 1, 2 MP

1. (A • B) ⊃ C
2. A • B / ∴ C
3. C 1, 2 MP

MP φ ⊃ ψ
φ
∴ ψ

1. (A • B) ⊃ C
2. A • B / ∴ C
3. C 1, 2 MP

MP
φ ⊃ ψ
φ
∴ ψ

Note: The greek statement variables can stand for simple or compound statements

---

1. P ⊃ ~Q
2. ~Q ⊃ ~P
3. ~P ⊃ (A ≡ K)
4. Q / ∴ A ≡ K

---

1. P ⊃ ~Q
2. ~Q ⊃ ~P
3. ~P ⊃ (A ≡ K)
4. Q / ∴ A ≡ K
5. ~P 1, 4 MT

---

1. P ⊃ ~Q
2. ~Q ⊃ ~P
3. ~P ⊃ (A ≡ K)
4. Q / ∴ A ≡ K
5. ~P 1, 4 MT

MT
φ ⊃ ψ
~ψ
∴ ~φ

Note: A tilde in the written rule means that the statement with the tilde must be the negation of that same statement without the tilde.

---

1. P ⊃ ~Q
2. ~Q ⊃ ~P
3. ~P ⊃ (A ≡ K)
4. Q / ∴ A ≡ K
5. ~P 1, 4 MT

MT
φ ⊃ ψ
~ψ
∴ ~φ

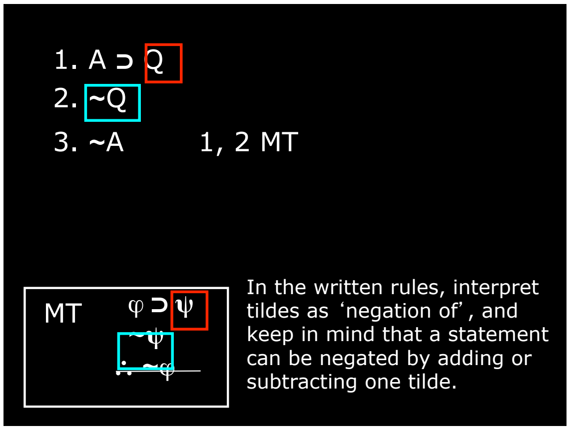
This means that one must be the negation of the other. But which one has the actual tilde on it doesn’t matter.

---

1. P ⊃ ~Q
2. ~Q ⊃ ~P
3. ~P ⊃ (A ≡ K)
4. Q / ∴ A ≡ K
5. ~P 1, 4 MT

MT
φ ⊃ ψ
~ψ
∴ ~φ

In the written rules, interpret tildes as ‘negation of’, and keep in mind that a statement can be negated by adding or subtracting one tilde.


1. A ⊃ Q
2. ~Q
3. ~A 1, 2 MT
MT φ ⊃ ψ
~ψ
∴ ~φ
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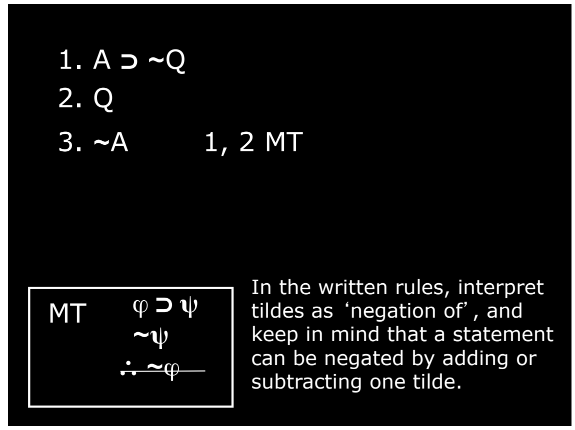
1. A ⊃ ~Q
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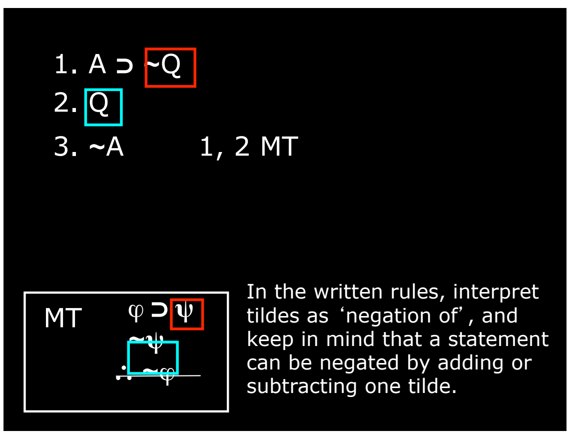
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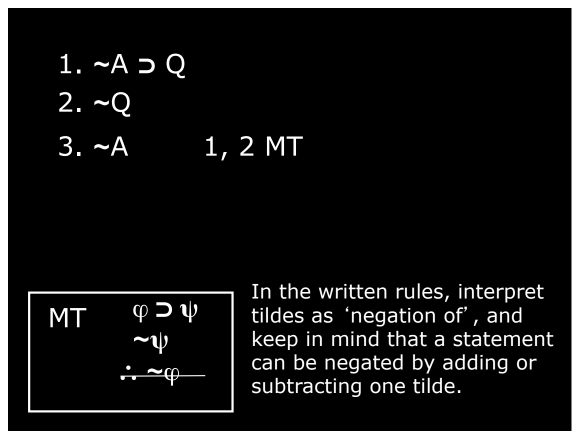
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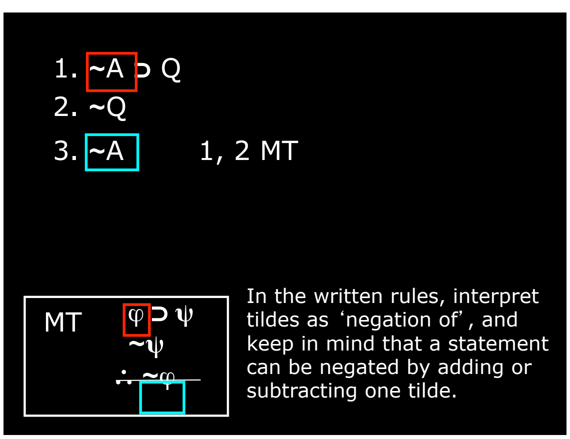
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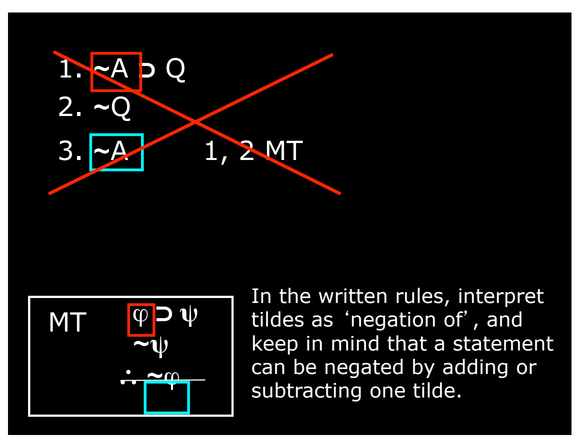
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1. ~A ⊃ Q
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In the written rules, interpret tildes as 'negation of', and keep in mind that a statement can be negated by adding or subtracting one tilde.

---

1. ~A ⊃ ~Q
2. Q
3. A 1, 2 MT

MT φ ⊃ ψ
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In the written rules, interpret tildes as 'negation of', and keep in mind that a statement can be negated by adding or subtracting one tilde.

---

1. ~A ⊃ ~Q
2. Q
3. A 1, 2 MT

MT φ ⊃ ψ
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In the written rules, interpret tildes as 'negation of', and keep in mind that a statement can be negated by adding or subtracting one tilde.

---

1. P ⊃ ~Q
2. ~Q ⊃ ~P
3. ~P ⊃ (A ≡ K)
4. Q / ∴ A ≡ K

5. ~P 1, 4 MT

---

1. P ⊃ ~Q
2. ~Q ⊃ ~P
3. ~P ⊃ (A ≡ K)
4. Q / ∴ A ≡ K

5. ~P 1, 4 MT
6. A ≡ K 3, 5 MP

Three more Inference Rules

Disjunctive Syllogism (DS)

$$\frac{\varphi \vee \psi \quad \sim\varphi}{\therefore \psi}$$

Three more Inference Rules

Hypothetical Syllogism (HS)

$$\frac{\varphi \supset \psi \quad \psi \supset \chi}{\therefore \varphi \supset \chi}$$

Three more Inference Rules

Simplification (simp)

$$\frac{\varphi \bullet \psi}{\therefore \psi}$$

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E

---

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E
3. G 2 simp

---

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E
3. G 2 simp
4. ~D 2 simp

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E
3. G 2 simp
4. ~D 2 simp

Simp $\dfrac{\varphi \bullet \psi}{\therefore \psi}$

---

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E
3. G 2 simp
4. ~D 2 simp
5. E ⊃ ~G 4, 1 DS

---

1. D v (E ⊃ ~G)
2. ~D • G / ∴ ~E
3. G 2 simp
4. ~D 2 simp
5. E ⊃ ~G 4, 1 DS
6. ~E 3, 5 MT

---

1. (B ⊃ A) • (C ⊃ D)
2. ~B
3. B v ~K
4. ~(A ⊃ D) ⊃ K / ∴ B ⊃ D

---

1. (B ⊃ A) • (C ⊃ D)
2. ~B
3. B v ~K
4. ~(A ⊃ D) ⊃ K / ∴ B ⊃ D
5. ~K 2, 3 DS

---

1. (B ⊃ A) • (C ⊃ D)
2. ~B
3. B v ~K
4. ~(A ⊃ D) ⊃ K / ∴ B ⊃ D
5. ~K 2, 3 DS
6. A ⊃ D 5, 4 MT

1. (B ⊃ A) • (C ⊃ D)
2. ~B
3. B ∨ ~K
4. ~(A ⊃ D) ⊃ K   / ∴ B ⊃ D
5. ~K   2, 3 DS
6. A ⊃ D   5, 4 MT
7. B ⊃ A   1 simp

---

1. (B ⊃ A) • (C ⊃ D)
2. ~B
3. B ∨ ~K
4. ~(A ⊃ D) ⊃ K   / ∴ B ⊃ D
5. ~K   2, 3 DS
6. A ⊃ D   5, 4 MT
7. B ⊃ A   1 simp
8. B ⊃ D   6, 7 HS

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Whole lines in the rule must correspond to whole lines on the proof.

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MT   φ ⊃ ψ
~ψ
~~∴ ~φ~~

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Whole lines in the rule must correspond to whole lines on the proof.

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MT: $\varphi \supset \psi$, $\sim\psi$, $\therefore \sim\varphi$

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2. ~Q
3. ~A v B   1, 2 MT

MT: φ ⊃ ψ, ~ψ, ∴ ~φ

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1. (A v B) ⊃ Q
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Don't distribute negation operators

1. (A v B) ⊃ Q
2. ~Q
3. ~A v ~B 1, 2 MT

What can't you do when applying inference rules?

Don't distribute negation operators

1. (A v B) ⊃ Q
2. ~Q
3. ~A v ~B 1, 2 MT

~A v ~B ≠ ~(A v B)

What can't you do when applying inference rules?

Don't distribute negation operators

~~1. (A v B) ⊃ Q~~
~~2. ~Q~~
~~3. ~A v ~B 1, 2 MT~~

~A v ~B ≠ ~(A v B)

What can't you do when applying inference rules?

Don't distribute negation operators

~~1. (A v B) ⊃ Q~~
~~2. ~Q~~
~~3. ~A v ~B 1, 2 MT~~

~A • ~B ≠ ~(A • B)

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1. $\sim A \supset Q$
2. $\sim Q$
3. A 1, 2 MT

MT: $\varphi \supset \psi$, $\sim\psi$, $\therefore \sim\varphi$

What can you do when applying inference rules?

The order of the lines in the rule need not be the same as the order of the lines in the proof (though of course the derived line must come after the lines it is derived from)

1. $\sim Q$
2. $\sim A \supset Q$
3. A 1, 2 MT

MT: $\varphi \supset \psi$, $\sim\psi$, $\therefore \sim\varphi$

What can you do when applying inference rules?

You can cite the lines in any order

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MT: $\varphi \supset \psi$, $\sim\psi$, $\therefore \sim\varphi$

What can you do when applying inference rules?

You can cite the lines in any order

1. $\sim Q$
2. $\sim A \supset Q$
3. A 2, 1 MT

MT: $\varphi \supset \psi$, $\sim\psi$, $\therefore \sim\varphi$

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For simplification, you can simplify either conjunct, and for disjunctive syllogism, you can have the negation of either disjunct

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For simplification, you can simplify either conjunct, and for disjunctive syllogism, you can have the negation of either disjunct

1. A
2. ~A v ~Q
3. ~Q 2, 1 DS

What can you do when applying inference rules?

For simplification, you can simplify either conjunct, and for disjunctive syllogism, you can have the negation of either disjunct

1. Q
2. ~A v ~Q
3. ~A 2, 1 DS

---

1. ~D v (H v H)
2. ~R • (H ⊃ ~ J)
3. (R • D) ⊃ ~J
4. (H ⊃ ~ J) ⊃ (H ⊃ R)
5. D / ∴ ~J

---

1. ~D v (H v H)
2. ~R • (H ⊃ ~ J)
3. (R • D) ⊃ ~J
4. (H ⊃ ~ J) ⊃ (H ⊃ R)
5. D / ∴ ~J
6. H ⊃ ~J 2 simp

---

1. ~D v (H v H)
2. ~R • (H ⊃ ~ J)
3. (R • D) ⊃ ~J
4. (H ⊃ ~ J) ⊃ (H ⊃ R)
5. D / ∴ ~J
6. H ⊃ ~J 2 simp
7. H v H 5, 1 DS

---

1. ~D v (H v H)
2. ~R • (H ⊃ ~ J)
3. (R • D) ⊃ ~J
4. (H ⊃ ~ J) ⊃ (H ⊃ R)
5. D / ∴ ~J
6. H ⊃ ~J 2 simp
7. H v H 5, 1 DS
8. H ⊃ R 6, 4 MP

1. ~D ∨ (H ∨ H)
2. ~R • (H ⊃ ~ J)
3. (R • D) ⊃ ~J
4. (H ⊃ ~ J) ⊃ (H ⊃ R)
5. D / ∴ ~J
6. H ⊃ ~J 2 simp
7. H ∨ H 5, 1 DS
8. H ⊃ R 6, 4 MP
9. ~R 2 simp

1. ~D ∨ (H ∨ H)
2. ~R • (H ⊃ ~ J)
3. (R • D) ⊃ ~J
4. (H ⊃ ~ J) ⊃ (H ⊃ R)
5. D / ∴ ~J
6. H ⊃ ~J 2 simp
7. H ∨ H 5, 1 DS
8. H ⊃ R 6, 4 MP
9. ~R 2 simp
10. ~H 9, 8 MT

1. ~D ∨ (H ∨ H)
2. ~R • (H ⊃ ~ J)
3. (R • D) ⊃ ~J
4. (H ⊃ ~ J) ⊃ (H ⊃ R)
5. D / ∴ ~J
6. H ⊃ ~J 2 simp
7. H ∨ H 5, 1 DS
8. H ⊃ R 6, 4 MP
9. ~R 2 simp
10. ~H 9, 8 MT
11. H 10, 7 DS

1. ~D ∨ (H ∨ H)
2. ~R • (H ⊃ ~ J)
3. (R • D) ⊃ ~J
4. (H ⊃ ~ J) ⊃ (H ⊃ R)
5. D / ∴ ~J
6. H ⊃ ~J 2 simp
7. H ∨ H 5, 1 DS
8. H ⊃ R 6, 4 MP
9. ~R 2 simp
10. ~H 9, 8 MT
11. H 10, 7 DS
12. ~J 6, 11 MP

1. ~(R ⊃ R) ∨ K
2. R ⊃ (S • K)
3. ~(S • A) ⊃ ~K
4. (S • K) ⊃ R
5. A ⊃ (B ⊃ ~K) / ∴ ~B

1. ~(R ⊃ R) ∨ K
2. R ⊃ (S • K)
3. ~(S • A) ⊃ ~K
4. (S • K) ⊃ R
5. A ⊃ (B ⊃ ~K) / ∴ ~B
6. R ⊃ R 2, 4 HS

1. ~(R ⊃ R) v K	
2. R ⊃ (S • K)	
3. ~(S • A) ⊃ ~K	
4. (S • K) ⊃ R	
5. A ⊃ (B ⊃ ~K)	/ ∴ ~B
6. R ⊃ R	2, 4 HS
7. K	6, 1 DS

1. ~(R ⊃ R) v K	
2. R ⊃ (S • K)	
3. ~(S • A) ⊃ ~K	
4. (S • K) ⊃ R	
5. A ⊃ (B ⊃ ~K)	/ ∴ ~B
6. R ⊃ R	2, 4 HS
7. K	6, 1 DS
8. S • A	7, 3 MT

1. ~(R ⊃ R) v K	
2. R ⊃ (S • K)	
3. ~(S • A) ⊃ ~K	
4. (S • K) ⊃ R	
5. A ⊃ (B ⊃ ~K)	/ ∴ ~B
6. R ⊃ R	2, 4 HS
7. K	6, 1 DS
8. S • A	7, 3 MT
9. A	8 simp

1. ~(R ⊃ R) v K	
2. R ⊃ (S • K)	
3. ~(S • A) ⊃ ~K	
4. (S • K) ⊃ R	
5. A ⊃ (B ⊃ ~K)	/ ∴ ~B
6. R ⊃ R	2, 4 HS
7. K	6, 1 DS
8. S • A	7, 3 MT
9. A	8 simp
10. B ⊃ ~K	9, 5 MP

1. ~(R ⊃ R) v K	
2. R ⊃ (S • K)	
3. ~(S • A) ⊃ ~K	
4. (S • K) ⊃ R	
5. A ⊃ (B ⊃ ~K)	/ ∴ ~B
6. R ⊃ R	2, 4 HS
7. K	6, 1 DS
8. S • A	7, 3 MT
9. A	8 simp
10. B ⊃ ~K	9, 5 MP
11. ~B	10, 7 MT

Final 3 Inference Rules

Dilemma (dil)

φ ⊃ ψ
χ ⊃ ξ
φ v χ
∴ ψ v ξ

Final 3 Inference Rules

Disjunction Introduction (DI)

$$\frac{\varphi}{\therefore\ \varphi \vee \psi}$$

Final 3 Inference Rules

Conjunction (conj)

$$\begin{array}{l} \varphi \\ \psi \\ \hline \therefore\ \varphi \bullet \psi \end{array}$$

1. (D v G) ⊃ (A v C)
2. D • (A ⊃ G)
3. C ⊃ H / ∴ D • (H v G)

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4. D 2 simp

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4. D 2 simp
5. D v G 4, DI

1. (D v G) ⊃ (A v C)
2. D • (A ⊃ G)
3. C ⊃ H / ∴ D • (H v G)
4. D 2 simp
5. D v G 4, DI
6. A v C 1, 5 MP

1. (D v G) ⊃ (A v C)	
2. D • (A ⊃ G)	
3. C ⊃ H	/ ∴ D • (H v G)
4. D	2 simp
5. D v G	4, DI
6. A v C	1, 5 MP
7. A ⊃ G	2 simp

1. (D v G) ⊃ (A v C)	
2. D • (A ⊃ G)	
3. C ⊃ H	/ ∴ D • (H v G)
4. D	2 simp
5. D v G	4, DI
6. A v C	1, 5 MP
7. A ⊃ G	2 simp
8. H v G	3, 6, 7 dil

1. (D v G) ⊃ (A v C)	
2. D • (A ⊃ G)	
3. C ⊃ H	/ ∴ D • (H v G)
4. D	2 simp
5. D v G	4, DI
6. A v C	1, 5 MP
7. A ⊃ G	2 simp
8. H v G	3, 6, 7 dil
9. D • (H v G)	4, 8 conj

1. (~A ⊃ ~C) ⊃ A	
2. C ⊃ (~A ⊃ ~C)	
3. ~A	/ ∴ ~C

1. (~A ⊃ ~C) ⊃ A	
2. C ⊃ (~A ⊃ ~C)	
3. ~A	/ ∴ ~C
4. ~(~A ⊃ ~C)	3, 1 MT

1. (~A ⊃ ~C) ⊃ A	
2. C ⊃ (~A ⊃ ~C)	
3. ~A	/ ∴ ~C
4. ~(~A ⊃ ~C)	3, 1 MT
5. ~C	2, 4 MT

1. (~A ⊃ ~C) ⊃ A
2. C ⊃ (~A ⊃ ~C)
3. ~A / ∴ ~C
4. ~(~A ⊃ ~C) 3, 1 MT
5. ~C 2, 4 MT

4. C ⊃ A 1, 2 HS

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1. (~A ⊃ ~C) ⊃ A
2. C ⊃ (~A ⊃ ~C)
3. ~A / ∴ ~C
4. ~(~A ⊃ ~C) 3, 1 MT
5. ~C 2, 4 MT

4. C ⊃ A 1, 2 HS
5. ~C 3, 4 MT

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1. (R v G) ⊃ ~A
2. A v W
3. R • S / ∴ W v X

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1. (R v G) ⊃ ~A
2. A v W
3. R • S / ∴ W v X
4. R 3 simp

---

1. (R v G) ⊃ ~A
2. A v W
3. R • S / ∴ W v X
4. R 3 simp
5. R v G 4 DI

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1. (R v G) ⊃ ~A
2. A v W
3. R • S / ∴ W v X
4. R 3 simp
5. R v G 4 DI
6. ~A 1, 5 MP

1. (R v G) ⊃ ~A
2. A v W
3. R • S / ∴ W v X
4. R 3 simp
5. R v G 4 DI
6. ~A 1, 5 MP
7. W 6, 2 DS

1. (R v G) ⊃ ~A
2. A v W
3. R • S / ∴ W v X
4. R 3 simp
5. R v G 4 DI
6. ~A 1, 5 MP
7. W 6, 2 DS
8. W v X 7 DI

1. M ⊃ D
2. G ⊃ M
3. G v G / ∴ D v M

1. M ⊃ D
2. G ⊃ M
3. G v G / ∴ D v M
4. G ⊃ D 1, 2 HS

1. M ⊃ D
2. G ⊃ M
3. G v G / ∴ D v M
4. G ⊃ D 1, 2 HS
5. D v M 2, 3, 4 dil

Conditional Proof

Conditional Proof

First of two proof strategies using 'subproofs'

Basic idea: prove the conditional $\varphi \supset \psi$ is true, by assuming φ and deriving ψ.

Conditional Proof

At some point in a proof, you decide you'd like to be able to derive $\varphi \supset \psi$ on a line, but you can't figure out how. Add an assumption line consisting of φ, then proceed using the rules.

Conditional Proof

Keep deriving lines until you derive ψ. At this point, we don't know whether φ is actually true, since we just assumed it, nor do we know whether ψ is true, but we have shown that if φ were true, then ψ would be true.

Conditional Proof

But this fact that the subproof demonstrated, that if φ were true, then ψ would be true, just is what the conditional $\varphi \supset \psi$ means. So the subproof shows that the conditional can be validly inferred.

Rules for the use of CP

1. Start subproof (SP) by indenting and designating first line ACP
2. CP ends any time you want
3. Mark off CP, closing SP and discharging assumption
4. The next line after the closed CP SP can only be a conditional whose antecedent is the ACP and whose consequent is the last line of the CP SP

Rules for the use of CP

5. All SPs must be closed before the proof can end
6. Once a SP has been closed, no lines in it may be used or cited

01. $G \supset T$
02. $(T \vee S) \supset K$ / $\therefore G \supset K$

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03. $\longrightarrow$ G ACP

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04. T 1, 3 MP
05. $T \vee S$ 4 DI

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02. $(T \vee S) \supset K$ / $\therefore G \supset K$
03. $\longrightarrow$ G ACP
04. T 1, 3 MP
05. $T \vee S$ 4 DI
06. K 2, 5 MP

01. $G \supset T$
02. $(T \vee S) \supset K$ / $\therefore G \supset K$
03. $\longrightarrow$ G ACP
04. T 1, 3 MP
05. $T \vee S$ 4 DI
06. K 2, 5 MP

01.	G ⊃ T	
02.	(T v S) ⊃ K	/ ∴ G ⊃ K
03.	→ G	ACP
04.	T	1, 3 MP
05.	T v S	4 DI
06.	K	2, 5 MP
07.	G ⊃ K	3-6 CP

01.	C ⊃ (A • D)	
02.	B ⊃ (A • E)	/ ∴ ~(C v B) v A

01.	C ⊃ (A • D)	
02.	B ⊃ (A • E)	/ ∴ ~(C v B) v A
03.	→ C v B	ACP

01.	C ⊃ (A • D)	
02.	B ⊃ (A • E)	/ ∴ ~(C v B) v A
03.	→ C v B	ACP
04.	(A • D) v (A • E)	1, 2, 3 dil

01.	C ⊃ (A • D)	
02.	B ⊃ (A • E)	/ ∴ ~(C v B) v A
03.	→ C v B	ACP
04.	(A • D) v (A • E)	1, 2, 3 dil
05.	A • (D v E)	4 dist

01.	C ⊃ (A • D)	
02.	B ⊃ (A • E)	/ ∴ ~(C v B) v A
03.	→ C v B	ACP
04.	(A • D) v (A • E)	1, 2, 3 dil
05.	A • (D v E)	4 dist
06.	A	5 simp

01. C ⊃ (A • D)
02. B ⊃ (A • E)   / ∴ ~(C v B) v A
03. → C v B   ACP
04. (A • D) v (A • E)   1, 2, 3 dil
05. A • (D v E)   4 dist
06. A   5 simp

01. C ⊃ (A • D)
02. B ⊃ (A • E)   / ∴ ~(C v B) v A
03. → C v B   ACP
04. (A • D) v (A • E)   1, 2, 3 dil
05. A • (D v E)   4 dist
06. A   5 simp
07. (C v B) ⊃ A   3-6 CP

01. C ⊃ (A • D)
02. B ⊃ (A • E)   / ∴ ~(C v B) v A
03. → C v B   ACP
04. (A • D) v (A • E)   1, 2, 3 dil
05. A • (D v E)   4 dist
06. A   5 simp
07. (C v B) ⊃ A   3-6 CP
08. ~(C v B) v A   7 CE

01. C ⊃ (A • D)
02. B ⊃ (A • E)   / ∴ ~(C v B) v A
03. → C v B   ACP
04. (A • D) v (A • E)   1, 2, 3 dil
05. A • (D v E)   4 dist
06. (C v B)⊃[A•(D v E)]   3-5 CP