EPR paradox and Bell’s theorem

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Einstein tried to prove that QM did not give a complete description of reality, using thought experiments involving various contraptions.

“For example, one of Einstein’s thought experiments involved a beam of electrons that is sent through a slit in a screen, and then the position of the electrons are recorded as they hit a photographic plate. Various other elements, such as a shutter to open and close the slit instantaneously, were posited by Einstein in his ingenious efforts to show that position and momentum could in theory be known with precision.

‘Einstein would bring along to breakfast a proposal of this kind,’ Heisenberg recalled...
“The group would usually make their way to the Congress hall together, working on ways to refute Einstein’s problem. ‘By dinner-time we could usually prove that his thought experiment did not contradict uncertainty relations,’ Heisenberg recalled, and Einstein would concede defeat. ‘But the next morning he would bring along to breakfast a new thought experiment, generally more complicated than the previous one.’ By dinnertime that would be disproved as well.

“Back and forth they went, each lob from Einstein volleyed back by Bohr, who was able to show how the uncertainty principle, in each instance, did indeed limit the amount of knowable information about a moving electron. ‘And so it went for several days,’ said Heisenberg. ‘In the end, we—that is, Bohr, Pauli, and I—knew that we could now be sure of our ground.’ ” (p. 346)
Einstein-Podolsky-Rosen 1935


The EPR-paradox reconstructed

- A source creates spin-1/2-particles (such as e\(^-\)) in a singlet state
  \[ |\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \]
  which are then separated s.t. one e\(^-\) moves to left wing, and the other to the right wing.

- Important: spins cancel, total spin is zero

\[ \Rightarrow \] If L particle is found in “up” state, then R particle must be in “down” state (and vice versa).

- In classical physics, that would not be a problem, since we would just conclude that R particle always had spin “down” from the time of separation.
However, according to (the standard interpretation of) QM, the spin of the L particle has no definite value until measured.

⇒ When it is measured, it must produce an instantaneous effect in R wing, collapsing the wave fct s.t. the R particle has definite spin too.

⇒ either spooky action-at-a-distance or faster-than-light signalling (⇒ violation of special relativity)

EPR: this shows that there must be hidden elements of reality ("hidden variables"), which QM fails to take into account, i.e. QM state description is incomplete
Locality

**Principle (Einstein locality)**

*If two systems are in isolation from each other s.t. they don't interact anymore, then a measurement on the first does not have any real effect on the second.*

- Bohr: Einstein locality is violated, the QM-system consists of both particles (and the observer), until a measurement is made

⇒ EPR-paradox doesn’t show that QM is incomplete, but only that Einstein locality is violated
Reminder: EPR tried to argue for the incompleteness of QM
 ⇒ Idea that there exists a “hidden reality” behind what is captured in the QM-description
 ⇒ David Bohm (1917-1992), in his Quantum Theory (1951), formulated a (nonlocal) hidden variable (HV) theory that was empirically equivalent to QM
⇒ In this work, Bohm extended the EPR thought experiment
⇒ ignited the interest of John S Bell (1928-1990)
John Stewart Bell (1928-1990)

- studied physics at Queen’s University Belfast, PhD U Birmingham, CERN
- “On the Einstein-Podolsky-Rosen paradox” (1964): derivation of Bell’s inequality
- Bell’s theorem: this inequality, derived from basic assumptions about locality and separability, conflicts with the predictions of QM
Bell’s relevance

By the mid-60s, almost all physicists just moved on and worked with QM, but didn’t reflect its foundations.

⇒ many of them didn’t notice, and still fail to appreciate the relevance of Bell’s thm

But not all: “Bell’s theorem is the most profound discovery of science” (Henry Stapp)

A bit more nuanced (but only a bit): “Anybody who’s not bothered by Bell’s theorem has to have rocks in his head” (“a distinguished Princeton physicist”)

Mermin’s classification of physicists:

- Type 1 bothered by EPR and Bell’s thm, type 2 (the majority) not bothered
- Type 2a explain why not, but either miss the point entirely or make assertions that are demonstrably false
- Type 2b refuse to explain why they are not bothered
Mermin’s version of the EPR-Bohm thought experiment

- Three pieces: two detectors (A and B), and a source (C)
- Each detector has switch with three settings (1, 2, 3), and responds to event by flashing red light (R) or green (G)
- No connections bw pieces \(\Rightarrow\) no signals other than particles
- Switch of each detector is independently and randomly set to one of its settings, and button is pushed at source to initiate process of creating pair and sending them to opposite wings
- many runs of the experiments are made, data of form (11GG, 23GR, etc) collected
Note: since there are no connection between parts of apparatus, the only thing that travels between them are the particles (this can be tested by sliding walls, etc)

The data has two features:

1. For those runs when settings were the same in A and B, we find that the light always flashed in the same colour. (PERFECT CORRELATION)

2. For all runs regardless of the settings in A and B, the pattern of flashing is completely random. In particular, half of the time the same colour flashes, half of the time a different one does. (NO CORRELATION)
How can this data be explained?

- perfect correlation cries out for explanation

- Traditional possibilities: events are really parts of one larger event, or A causes B or vice versa, or they have common cause

- If detectors could communicate, this would be easy. But they don’t. And can’t.

- Neither can the detectors have been preprogrammed always to flash same colour, since they also need to account for data point 2, and their settings are random and independent.

- Born offers an explanation (in a letter of May 1948 to Einstein): “objects far apart in space which have a common origin need not be independent... Dirac has based his whole book on this.”

- Mermin makes this more concretely on p. 43f, let’s look at this
At core a common cause expl: both particles are imparted the same ordered triple of labels as they leave the source (three bits of information, e.g. RRG, GRG, etc), each telling the detector which colour to flash, depending on its setting.

Mermin imagines a possibility: particles come in eight different kinds (cubes, spheres, tetrahedra, etc), but this is essentially the same idea: each particle carries with it a set of instructions for how to flash for each of the three settings, and that in any run both particles carry the same set of instructions.

Instructions must cover each of the possible detector settings bc there is no communication bw source and detectors other than the particles.

This also means that instructions must be carried in every run, since one can never know at the source whether the settings are the same.

⇒ can easily account for data 1
But despite the naturalness of this type of explanation (arguably the only natural expl), it cannot be true: it’s inconsistent with data 2!

Note that “we are about to show that ‘something one cannot know anything about’—the third entry in an instruction set—cannot exist.” (43) (one can never learn more than two of the entries in the instruction sets imparted on the particles)

Here’s the arg for the inconsistency w/ data 2. Consider a possible instruction set, e.g. RRG.

⇒ detectors will flash same colour for settings 11, 22, 33, 12, 21, and different colour for settings 13, 31, 23, 32

⇒ since settings are random and independent, each of the nine possibilities are equally probable

⇒ instruction set RRG will result in same colour flashing in 5/9 of the time
Evidently, the same holds for instruction sets RGR, GRR, GGR, GRG, and RGG (bc arg uses only the fact that one colour appears twice, and the other once).

Two more instruction sets are left: RRR and GGG, but these both result in the same colours flashing all the time (w/ probability one). But this gives us the famous:

Theorem (Bell’s theorem (baby version))

*If instruction sets exist, the same colours will flash in at least \( \frac{5}{9} \) of all the runs, regardless of how the instruction sets are distributed among the runs.*

- This is **Bell’s inequality (baby version)**: the probability that the same colours flash is larger or equal to \( \frac{5}{9} \).
- It’s now obvious that data 2 cannot be accounted for: data 2 violates Bell’s inequality!

⇒ there cannot be a local hidden variable explanation
Comments

- simplified thought experiment exactly captures the relevant features of the EPR-Bohm experiment, except that it introduces runs where the orientations in both wings are not aligned.

- Baby Bell theorem shows why there cannot be local hidden vars, contra EPR who argued that QM was incomplete.

- Bell was the one who added the runs with different settings in order to extract from QM the prediction about data 2.

- It was exactly data 2 that showed that a local HV story is incompatible with the predictions of QM.

- Alain Aspect, Paris 1982; Nicolas Gisin, Geneva 1997: detectors are 10 km apart, settings chosen after photons left source.

⇒ experimental falsification of local HV thy
Non-locality is not just an artifact of standard QM

Albert, Quantum Mechanics and Experience, Ch. 3.

- EPR thought that the nonlocal character of measurements on non-separable states is a merely disposable artifact of the particular formalism of standard QM.

- The upshot of Bell’s thm is that this is demonstrably wrong:

  “What Bell has given us is a proof that there is as a matter of fact a genuine nonlocality in the actual workings of nature, however we attempt to describe it, period. That nonlocality is... necessarily... a feature of every possible manner of calculating... which produces the same statistical predictions as quantum mechanics does; and those predictions are now experimentally known to be correct.” (70)
Three final comments


Three results concerning the “quantum connection”:

1. **It is unattenuated**: in contrast to classical (instantaneous) action, the quantum connection is unaffected by distance.

2. **It is discriminating**: while gravitational forces affect similarly situated objects in the same way, the quantum connection is a private arrangement between entangled particles.

3. **It is instantaneous**: while Newton’s theory of gravity has gravity propagate instantaneously, it need not do so, and GR certainly involves no instantaneous gravitational action; but the quantum connection appears to act essentially instantaneously.